MATHEMATICS COURSE SYLLABUS

Course Title: Advanced Algebra H2 (Spring 2014 Revision)

Department: Mathematics

Primary Course Materials:
Textbook: Algebra 2
Publisher: Prentice Hall
Author: Bellman, Bragg, Charles, Handlin, Kennedy

Supplementary Materials
TI-Nspire Graphing Calculator Applications

Course Description:
This course is specifically designed for students who have successfully completed Intermediate algebra. This course can be taken simultaneously with Geometry. All students will be actively engaged in problem solving, reasoning, connecting and communicating mathematically as they study the following topics: functions and relations; matrices and determinants; quadratic functions and complex numbers; polynomial functions; powers, roots and radicals; rational expressions, equations, and functions; and (time permitting) combinatorics and the Binomial Theorem.

Students will be required to keep an organized notebook, read and interpret the textbook, and do some independent work. Emphasis will be placed on investigating and solving real-world problems that will include open-ended and open-response questions.

Since this course will advocate and encourage the proper use of technology, the purchase of a TI-84+ or TI-Nspire graphing calculator is strongly recommended.

Essential Questions:
1. How can matrices be used to organize data and solve systems of equations?
2. How are functions used to analyze and model real world applications in a variety of disciplines, such as science, business, and economics?
3. How can technology be used to enhance the understanding of function families and algebraic concepts?

Course Objectives:

Common Goals:
Thinking and Communicating
1) ☑ Read information critically to develop understanding of concepts, topics and issues.
2) ☑ Write clearly, factually, persuasively and creatively in Standard English.
3) ☑ Speak clearly, factually, persuasively and creatively in Standard English.
4) ☑ Use computers and other technologies to obtain, organize and communicate information and to solve problems.
5) ☑ Conduct research to interpret issues or solve complex problems using a variety of data and information sources.

Gain and Apply Knowledge in and across the Disciplines
6) Gain and Apply Knowledge in:
a) ☑ Literature and Language
b) ☑ Mathematics
c) Science and Technology  
d) Social Studies, History and Geography  
e) Visual and Performing Arts  
f) Health and Physical Education

Work and Contribute  
7) Demonstrate personal responsibility for planning one’s future academic and career options.  
8) Participate in a school or community service activity.  
9) Develop informed opinions about current economic, environmental, political and social issues affecting Massachusetts, the United States and the world and understand how citizens can participate in the political and legal system to affect improvements in these areas.

Learning Standards from the Massachusetts Curriculum Framework:  
A chart is attached identifying which of the standards from the Massachusetts Curriculum Frameworks will be assessed in this course.

21st Century Skills:

<table>
<thead>
<tr>
<th>Instructional practices support the achievement of 21st C. Learning Expectations by:</th>
<th></th>
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<tbody>
<tr>
<td>1. Demonstrate transliteracy by communicating across a range of platforms, tools and media.</td>
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<td>2. Utilize real-world digital tools and other resources to access, evaluate and share information in an authentic task.</td>
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<td>3. Demonstrate innovation, flexibility and adaptability in thinking patterns, work habits and working/learning conditions.</td>
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<td>4. Work independently and collaboratively to solve problems and accomplish goals.</td>
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<td>5. Value and demonstrate personal responsibility, ethical behavior and global awareness in both academic and social communities.</td>
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Content Outline:

Section 1: Linear Equations and Functions Review  
- Function notation and definition  
- Domain and range  
- Graphing lines (y=mx+b and ax+by=c)  
- Solving systems  
  - Substitution, linear combination, graphing  

Section 2: Matrices  
- Using matrices to represent data  
- Addition and subtraction of matrices  
- Matrix multiplication  
- 2x2 and 3x3 determinants  
- 2x2 and 3x3 identity and inverse matrices  
- Writing linear systems using matrices  
- Solving linear systems using matrices  
  - Inverse approach  
  - Cramer’s Rule  
  - By-hand and calculator methods  

6 days  
20 days
Section 3: Quadratic Functions

- Modeling data with quadratic functions
- Graphing
  - Standard and vertex forms
- Factoring quadratic expressions
  - Greatest common factors (GCFs), when the leading coefficient is equal to 1 or not equal to 1, splitting the middle term, difference of two squares, perfect square trinomials
- Solving quadratic equations by factoring, using the graphing calculator, or taking square roots
- Simplifying radicals
- Complex Numbers
  - Addition, subtraction, multiplication, division, absolute value, plotting in the complex plane
- Solving quadratic equations by completing the square or using the Quadratic Formula
- The discriminant and its use to determine the number and type of solutions
- Applications
  - Falling object, revenue and area word problems

Section 4: Higher Degree Polynomials

- Introduction to polynomial functions
  - Definition
  - Classifying by degree and number of terms
  - Simplifying polynomials and writing them in standard form
- Greatest common factors (GCFs), linear factors, and zeros
- Graphing polynomials by hand
  - End behavior, zeros, relative extrema
- Long division and synthetic division
- Remainder Theorem
- Special factoring cases
  - Sum and difference of cubes, factoring using quadratic form
- Theorems about the roots of polynomials
  - Rational Root Theorem, Irrational Root Theorem, Imaginary Root Theorem, Fundamental Theorem of Algebra
- Finding all the zeros of a polynomial function
- Graphing calculator exercises
  - Solving polynomial equations by graphing
  - Creating and comparing models fit to data
  - Word problems (maximizing volume)

Section 5: Radical Functions & More with Functions

- Properties of exponents
- Roots and radical expressions
  - Simplifying nth roots
  - Roots and exponents as inverses
- Multiplying and dividing radical expressions
- Adding and subtracting radical expressions
  - Operations with binomial radical expressions
- Rational exponents
  - Converting between radical and exponential form
  - Using the properties of exponents to simplify radical expressions
- Solving radical equations (including rational exponents)
  - Finding extraneous solutions
- Function operations
  - Addition, subtraction, multiplication, division, and composition of functions
• Inverse functions
  o Finding function inverses algebraically and graphically
  o Domain and range
  o Composition of inverse functions
• Graphing radical functions (square root and cube root)
  o Transformations

**Section 6: Rational Functions**
10 days
• Definition
• Graphing
  o Points of discontinuity (holes)
  o Vertical and horizontal asymptotes
• Simplifying rational expressions
• Multiplying and dividing rational expressions
• Adding and subtracting rational expressions

**Section 7: Permutations and Combinations (**Time Permitting**)**
8 days
• Definition, formulas, fundamental counting principle
• Word problems
• Expanding binomial expressions
  o Binomial Theorem, Pascal’s triangle
## High School Content Standards

### Conceptual Category: Number and Quantity

<table>
<thead>
<tr>
<th>N-RN</th>
<th>The Real Number System</th>
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<tbody>
<tr>
<td></td>
<td>Extend the properties of exponents to rational exponents.</td>
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<tr>
<td></td>
<td>1. Explain how the definition of the meaning of rational exponent follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define (5^{1/3}) to be the cube root of 5 because we want ((5^{1/3})^3 = 5^{1/3} \times 3) to hold, so ((5^{1/3})^3) must equal 5.</td>
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<td>2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.</td>
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<td></td>
<td>Use properties of rational and irrational numbers.</td>
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<td>3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.</td>
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<table>
<thead>
<tr>
<th>N-Q</th>
<th>Quantities</th>
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<tbody>
<tr>
<td></td>
<td>Reason quantitatively and use units to solve problems.</td>
</tr>
<tr>
<td></td>
<td>1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. *</td>
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<td></td>
<td>2. Define appropriate quantities for the purpose of descriptive modeling. ★</td>
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<td></td>
<td>3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★</td>
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<td></td>
<td>MA.3.a. Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure. ★</td>
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<tr>
<th>N-CN</th>
<th>The Complex Number System</th>
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<tr>
<td></td>
<td>Perform arithmetic operations with complex numbers.</td>
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<tr>
<td></td>
<td>1. Know there is a complex number (i) such that (i^2 = -1), and every complex number has the form (a + bi) with (a) and (b) real.</td>
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<td></td>
<td>2. Use the relation (i^2 = -1) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</td>
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<td></td>
<td>3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.</td>
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</table>

* indicates Modeling standard.
(+ ) indicates standard beyond College and Career Ready.
Represent complex numbers and their operations on the complex plane.

4. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, \((-1 + \sqrt{3}i)^2 = 8\) because \((-1 + \sqrt{3}i)\) has modulus 2 and argument 120°.

5. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

6. Solve quadratic equations with real coefficients that have complex solutions.

7. (+) Extend polynomial identities to the complex numbers. For example, rewrite \(x^2 + 4\) as \((x + 2i)(x - 2i)\).

8. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities

Perform operations on matrices and use matrices in applications.

1. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

2. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

3. (+) Add, subtract, and multiply matrices of appropriate dimensions.

4. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

5. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

Conceptual Category: Algebra

Seeing Structure in Expressions

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context. *
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret \(P(1 + r)^n\) as the product of \(P\) and a factor

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not depending on $P$.

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

### Arithmetic with Polynomials and Rational Expressions

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Understand the relationship between zeros and factors of polynomials.

Understand the relationship between zeros and factors of polynomials.

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

4. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal’s Triangle.$^1$

Rewrite rational expressions.

5. Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $\frac{q(x)}{b(x)} + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

### Creating Equations

$^1$ The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.
<table>
<thead>
<tr>
<th>CE D</th>
<th>Create equations that describe numbers or relationships.</th>
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<tbody>
<tr>
<td>1.</td>
<td>Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. *</td>
</tr>
<tr>
<td>2.</td>
<td>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td>
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<tr>
<td>3.</td>
<td>Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</td>
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<tr>
<td>4.</td>
<td>Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R.</td>
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<table>
<thead>
<tr>
<th>A-REI</th>
<th>Reasoning with Equations and Inequalities</th>
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<tbody>
<tr>
<td>1.</td>
<td>Understand solving equations as a process of reasoning and explain the reasoning.</td>
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<tr>
<td>2.</td>
<td>Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</td>
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<tr>
<td>3.</td>
<td>Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</td>
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<tr>
<td>4.</td>
<td>Solve equations and inequalities in one variable.</td>
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<tr>
<td></td>
<td>MA.3.a. Solve linear equations and inequalities in one variable involving absolute value.</td>
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<td></td>
<td>MA.4.c. Demonstrate an understanding of the equivalence of</td>
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</tbody>
</table>

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factoring, completing the square, or using the quadratic formula to solve quadratic equations.

Solve systems of equations.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. **Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line** $y = -3x$ **and the circle** $x^2 + y^2 = 3$.
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).

Represent and solve equations and inequalities graphically.
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

**Conceptual Category: Functions**

**F-IF** Understand the concept of a function and use function notation.
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.
3. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **Key features include:** intercepts; intervals where the function is increasing, decreasing, positive, or negative;

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4. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.*

**Analyze functions using different representations.**

6. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

7. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

MA.8.c. Translate among different representations of functions and relations: graphs, equations, point sets, and tables.

8. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

9. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.

### Building Functions

**Build a function that models a relationship between two quantities.**

1. Write a function that describes a relationship between two quantities.*

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2. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

3. Find inverse functions.
   a. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.

<table>
<thead>
<tr>
<th>S-ID</th>
<th>Conceptual Category: Statistics &amp; Probability</th>
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<tbody>
<tr>
<td></td>
<td><strong>Interpreting Categorical and Quantitative Data</strong></td>
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<td></td>
<td><strong>Summarize, represent, and interpret data on two categorical and quantitative variables.</strong></td>
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<tr>
<td>5.</td>
<td>Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</td>
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<tr>
<td></td>
<td>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.</td>
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<tr>
<td></td>
<td>c. Fit a linear function for a scatter plot that suggests a linear association.</td>
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