MATHEMATICS COURSE SYLLABUS

Course Title: College Algebra and Trigonometry

Department: Mathematics

- **Primary Course Materials:** Precalculus – A Graphing Approach (HOLT)
- Teacher designed materials to supplement the textbooks

Course Description:

This course is designed to provide a foundation and extensive study of topics focusing on exponential, logarithmic, and trigonometric functions. Other topics included in the course are radicals, complex numbers, rational functions, and the equation of a circle. A TI-84 graphing calculator will be used throughout the course as a tool to enhance conceptual understanding. The students will receive instruction and practice in the construction and interpretation of the function graphs studied in this course.

The instructional format will include large and small group instruction, question and answer sessions, lecture, lab and project activities. Presentation of new content will be through real world context and problem solving. Concepts will be presented with multiple representations; numerically, graphically, symbolically, and verbally.

The students will develop skills specific to the study of mathematics and develop test taking skills necessary for standardized tests and college placement exams.

Essential Questions:

1. How do the terms and symbols that are unique to algebra and trigonometry enable us to read, write, speak about, and understand mathematics?
2. How do we use exponential, logarithmic, and trigonometric functions to justify our conclusions and solutions to real world problems?
3. How do we use technology to analyze data and develop mathematical models?
4. How do we use a graphing calculator and other technologies to understand the nature and behavior of exponential, logarithmic, and trigonometric functions?

Common Goals:

Thinking and Communicating

1) ☒ Read information critically to develop understanding of concepts, topics and issues.
2) ☒ Write clearly, factually, persuasively and creatively in Standard English.
3) ☒ Speak clearly, factually, persuasively and creatively in Standard English.
4) ☒ Use computers and other technologies to obtain, organize and communicate information and to solve problems.
5) ☒ Conduct research to interpret issues or solve complex problems using a variety of data and information sources.

Gain and Apply Knowledge in and across the Disciplines

6) Gain and Apply Knowledge in:
   a) ☐ Literature and Language
   b) ☒ Mathematics
   c) ☒ Science and Technology
   d) ☐ Social Studies, History and Geography
   e) ☐ Visual and Performing Arts
   f) ☐ Health and Physical Education

Work and Contribute

7) ☒ Demonstrate personal responsibility for planning one's future academic and career options.
8) ☐ Participate in a school or community service activity.

9) ☐ Develop informed opinions about current economic, environmental, political and social issues affecting Massachusetts, the United States and the world and understand how citizens can participate in the political and legal system to affect improvements in these areas.

Learning Standards from the Massachusetts Curriculum Framework:

A chart is attached identifying which of the standards from the Massachusetts Curriculum Frameworks will be assessed in this course.

Additional Learning Objectives Beyond the Curriculum Framework:

Other Resources:

- TI-84 Graphing Calculator
- Software in Math Lab Rm. 344
  - Graphical Analysis
  - Microsoft Word
  - Microsoft Excel
  - Geometer's Sketchpad

21st Century Skills:

Instructional practices support the achievement of 21st C. Learning Expectations by: *(check those that apply to the Unit)*	__X___ personalizing instruction __X___ engaging students in cross disciplinary learning __X___ engaging students as active and self directed learners __X___ emphasizing inquiry, problem solving and higher order thinking __X___ applying knowledge and skills in authentic tasks __X___ engaging students in self assessment and reflection __X___ integrating technology

Content Outline:

1. **Functions** **(15 days)**
   a. What is a Function? (Domain, Range, interval notation)
   b. Graphing (vertical line test)
   c. Library of Functions (Odd, Even review)
   d. Transformations
   e. Composition
   f. Inverse of Linear Functions (verification numerically, graphically, algebraically)

2. **Quadratics** **(3 days)**
   a. Factoring
   b. Quadratic Formula

3. **Rational Functions** **(10 days)**
   a. Algebraically: Domain, y-intercepts, x-intercepts, vertical and horizontal asymptotes, holes, and zeroes.
   b. Graph: Verify.
   c. Simplify, Add, Subtract, Multiply, and Divide (with function notation)

4. Log and Exponential Functions (25 days)

a. Radicals and Rational Exponents
b. Definition of Exponential Functions (Compound Interest)
c. Growth and Decay Models (Applications)
d. Translate between Exponent and Log Expressions (Inverse, Compare Graphs)
e. Expand and Simplify (Properties and Laws of Logarithms)
f. Solve Logs (Solve Exponential Equations by using Logarithms)
g. Word Problems (Application)

5. Trigonometry (25 days)

a. SOH CAH TOA
    i. Converting DMS in Calculator
    ii. Calculator Use
b. Applications of SOH CAH TOA (Including Inverse Trig Functions)
c. Angle Measure (Degree and Radians, Application of Radian Measure)
d. Circles
    i. Graphing
    ii. Equations of
e. Unit Circle
    i. Reference Angles
    ii. Point Notation (Find Reference Angles given coordinate on Circle)
    iii. Derive Unit Circle using Special Right Triangles
    iv. Trig Expressions (All Quadrants)

6. Graphs of Sine and Cosine Functions (12 days)

a. Transformations
b. Period, Amplitude, Domain, Range
c. Develop Equation from Graph (Radians and Degrees)
d. Trigonometric Modeling using Graphing Calculator

7. Solving Triangles (10 days)

a. Law of Sines
b. Law of Cosines
c. Area of Triangles
d. Applications with Geometry

8. Trigonometry (4 days)

a. Solve Equations(Inverse Trigonometry)
b. Identities

Major Evaluation Strategies:

Name of Assessment	Type of Assessment		Common Goals Assessed	Standards Assessed	Other Objectives Assessed
	Test	Performance Assessment			
Oil Spill, Such a Deal	☐	☒	1,2,4,5,6b,6c		
FORE, Moon Rocks	☐	☒	1,2,4,5,6b,6c		
What's the Height?	☐	☒	1,2,3,4,5,6b,6c		
Daylight Around the World, Biorhythm Project	☐	☒	1,2,4,5,6b,6c		
You Tarzan… Me Jane, Ferris Wheel	☐	☒	1,2,4,5,6b,6c		
Population Changes, Financial Planning	☐	☒	1,2,4,5,6b,6c		
Test Quizzes Short Answer Multiple Choice Open-Ended Response	☒	☐	1,2,4,6b,6c		
SAT and SAT 2 Materials Accuplacer Materials	☒	☐	1,2,3,4,5		
Homework and Class Participation	☐	☒	1,2,3,4,5,6b,6c,7		
Midyear Exam	☒	☐	1,2,4,6b,6c		
Final Exam or Alternative Assessment	☒	☐	1,2,4,6b,6c		
	☐	☐			

High School Content Standards

Conceptual Category: Number and Quantity

N-RN The Real Number System

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponent follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

N-Q Quantities

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ⋆
2. Define appropriate quantities for the purpose of descriptive modeling. ★
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★
   MA.3.a. Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements. Identify significant figures in recorded measures and computed values based on the context given and the precision of the tools used to measure. ★

N-CN The Complex Number System

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Use complex numbers in polynomial identities and equations.

4. Solve quadratic equations with real coefficients that have complex solutions.

⋆ indicates Modeling standard.
(+) indicates standard beyond College and Career Ready.

	Conceptual Category: Algebra
A-SSE	**Seeing Structure in Expressions** **Interpret the structure of expressions.** 1. Interpret expressions that represent a quantity in terms of its context. ⋆ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1 + r)^n$ *as the product of P and a factor not depending on P.* 2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$. **Write expressions in equivalent forms to solve problems.** 3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines.
A-APR	**Arithmetic with Polynomials and Rational Expressions** 1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Understand the relationship between zeros and factors of polynomials. **Understand the relationship between zeros and factors of polynomials.** 2. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **Rewrite rational expressions.** 3. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
A-CED	**Creating Equations** **Create equations that describe numbers or relationships.** 1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear

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	and quadratic functions, and simple rational and exponential functions. ⋆ 2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ⋆ 3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ⋆ 4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.* ⋆
A-REI	**Reasoning with Equations and Inequalities** **Understand solving equations as a process of reasoning and explain the reasoning.** 1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. 2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **Solve equations and inequalities in one variable.** 3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. MA.3.a. Solve linear equations and inequalities in one variable involving absolute value. 4. Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. MA.4.c. Demonstrate an understanding of the equivalence of factoring, completing the square, or using the quadratic formula to solve quadratic equations. **Represent and solve equations and inequalities graphically.** 5. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

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	6. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.⋆
	Conceptual Category: Functions
F-IF	**Understand the concept of a function and use function notation.** 1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. 2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. **Interpret functions that arise in applications in terms of the context.** 3. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ⋆ 4. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ⋆ **Analyze functions using different representations.** 5. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ⋆ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. ⋆ b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. ⋆ c. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ⋆ d. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions,

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	showing period, midline, and amplitude. ⋆ 6. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, *and* $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* MA.8.c. Translate among different representations of functions and relations: graphs, equations, point sets, and tables. 7. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* 8. Given algebraic, numeric and/or graphical representations of functions, recognize the function as polynomial, rational, logarithmic, exponential, or trigonometric.
F-BF	**Building Functions** **Build a function that models a relationship between two quantities.** 1. Write a function that describes a relationship between two quantities.⋆ a. Determine an explicit expression, a recursive process, or steps for calculation from a context.⋆ b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.* ⋆ c. (+) Compose functions. *For example, if* $T(y)$ *is the temperature in the atmosphere as a function of height, and* $h(t)$ *is the height of a weather balloon as a function of time, then* $T(h(t))$ *is the temperature at the location of the weather balloon as a function of time.* ⋆ **Build new functions from existing functions.** 2. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic*

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	expressions for them. 3. Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x + 1)/(x - 1)$ *for* $x \neq 1$.
F-LE	**Linear, Quadratic, and Exponential Models** **Construct and compare linear, quadratic, and exponential models and solve problems.** 1. Distinguish between situations that can be modeled with linear functions and with exponential functions.⋆ a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. ⋆ b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. ⋆ c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. ⋆ 2. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.⋆ 3. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.⋆ **Interpret expressions for functions in terms of the situation they model.** 4. Interpret the parameters in a linear or exponential function in terms of a context. ⋆
F-TF	**Extend the domain of trigonometric functions using the unit circle.** 1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. 2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. 3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. 4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **Model periodic phenomena with trigonometric functions.** 5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.⋆

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	Prove and apply trigonometric identities. 6. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant. 7. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
G-CO	**Conceptual Category: Geometry**
G-SRT	**Similarity, Right Triangles, and Trigonometry** **Define trigonometric ratios and solve problems involving right triangles.** 1. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. 2. Explain and use the relationship between the sine and cosine of complementary angles. 3. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ⋆ **Apply trigonometry to general triangles.** 4. (+) Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. 5 . (+) Prove the Laws of Sines and Cosines and use them to solve problems. 6. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
G-MG	**Modeling with Geometry** **Apply geometric concepts in modeling situations.** 7. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★ MA.4. Use dimensional analysis for unit conversions to confirm that expressions and equations make sense. ★
	Conceptual Category: Statistics & Probability
S-ID	**Interpreting Categorical and Quantitative Data** **Interpret linear models.** 8. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ⋆

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	MODEL ADVANCED COURSE: Model Precalculus
N-CN	**The Complex Number System** **Perform arithmetic operations with complex numbers.** 1. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
F-IF	**Interpreting Functions** **Analyze functions using different representations.** 1. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ⋆ d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. ★
F-BF	**Building Functions** **Build a function that models a relationship between two quantities.** 1. Write a function that describes a relationship between two quantities. ⋆ c. (+) Compose functions. *For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.* ★ **Build new functions from existing functions.** 2. Find inverse functions. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. 3. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
F-TF	**Trigonometric Functions** **Extend the domain of trigonometric functions using the unit circle.** 1. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. 2. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

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	Prove and apply trigonometric identities. 3. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
G-SRT	**Similarity, Right Triangles, and Trigonometry** **Apply trigonometry to general triangles.** 1. (+) Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. 2. (+) Prove the Laws of Sines and Cosines and use them to solve problems. 3. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
G-C	**Circles**
	MODEL ADVANCED COURSE: Quantitative Reasoning
F-TF	**Trigonometric Functions** **Extend the domain of trigonometric functions using the unit circle.** 1. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number. 2. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **Model periodic phenomena with trigonometric functions.** 3. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ⋆ 4. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★ **Prove[1] and apply trigonometric identities.** 5. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
G-SRT	**Similarity, Right Triangles, and Trigonometry** **Apply trigonometry to general triangles.** 1. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

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[1] Advanced Quantitative Reasoning should accept informal proof and focus on the underlying reasoning, and use the theorems to solve problems.

G-C	**Circles** **Understand and apply theorems about circles.**
G-MG	**Modeling with Geometry** **Apply geometric concepts in modeling situations.** 1. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★ 2. Use dimensional analysis for unit conversions to confirm that expressions and equations make sense. ★